MATH 3339 Statistics for the Sciences

Formulas for Midterm - Mini Session

Descriptive Statistics

- $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_1$, sample mean
- $\tilde{x} = \begin{cases} \text{middle value of ordered data} & \text{if } n \text{ is odd} \\ \text{mean of two middle values of ordered data} & \text{if } n \text{ is even} \end{cases}$, sample median
- $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$, sample variance
- $s = \sqrt{s^2}$, sample standard deviation
- $IQR = Q_3 - Q_1$, interquartile range where Q_3 is the 75^{th} percentile and Q_1 is the 25^{th} percentile.
- $z = \frac{x-\bar{x}}{s}$, standard score
- $CV = \frac{s}{\bar{x}}$, coefficient of variation
- $cov(x, y) = \frac{1}{n-1}\sum_{i=1}^{n}(x_1 - \bar{x})(y_i - \bar{y})$, sample covariance
- $r = \frac{cov(x,y)}{s_x s_y}$, sample correlation
- $R^2 = r^2$, coefficient of determination
- $\hat{\beta}_1 = r\frac{s_y}{s_x}$, sample slope of least-squares regression line
- $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}$, sample y-intercept of least-squares regression line
- $y_i - \hat{y}_i$, residual of i^{th} observation

Counting and Probability

- $n! = n \cdot (n-1) \cdot (n-2) \cdots 1$, factorial
- $P(n, r) = \frac{n!}{(n-r)!}$, permutation
- $C(n, r) = \frac{n!}{r!(n-r)!}$, combination

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, general addition rule for probability
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$, conditional probability
- Given that events A_i, $i = 1, 2, \ldots, n$, are disjoint and exhaustive of the sample space S, then $P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i)$, law of total probability.

Discrete Probabilities

- $E(X) = \mu_x = \sum_{i=1}^{n} x_i p_i$, expected value of the distribution
- $Var(X) = \sigma_x^2 = \sum_{i=1}^{n} (x_i - \mu_x)^2 p_i = E(X^2) - E(X)^2$, variance of the distribution
- $SD(X) = \sigma_x = \sqrt{\sigma_x^2}$, standard deviation of the distribution
- **Binomial Distribution**, n = number of trials, p = probability of success
  - $P(X = k) = C(n, k)p^k(1 - p)^{n-k}$, probability of x successes in n independent trials
  - $E(X) = np$, expected value of binomial distribution
  - $Var(X) =$ np(1-p) , variance of binomial distribution
- **Geometric Distribution**, p = probability of success
  - $P(X = x) = (1 - p)^(x - 1)p$, probability that the x^{th} trial is the first success
  - $P(X > x) = (1 - p)^x$, probability that the first success is more than the x^{th} trial
  - $E(X) = \frac{1}{p}$ expected value of geometric distribution
  - $Var(X) = \frac{1-p}{p^2}$, variance of geometric distribution
- **Hypergeometric Distribution**, m = number of successes in the population, n = number of failures in the population, k = number of trails
  - $P(X = x) = \frac{C(m,x) \times C(n,k-x)}{C(m+n,k)}$, probability of x successes
  - $E(X) = \frac{km}{m+n}$, expected value of hypergeometric distribution
  - $Var(X) = kp(1 - p)\left(1 - \frac{k-1}{m=n-1}\right)$, variance of hypergeometric distribution, where $p = \frac{k}{m+n}$
- **Poisson Distribution**, μ = mean number of successes in a unit
  - $P(X = x) = \frac{e^{-\mu}\mu^x}{x!}$ probability of x successes
  - $E(X) = \mu$, expected value for Poisson distribution
  - $Var(X) = \mu$, variance for Poisson distribution

R Commands

- `factorial(n)`, factorial
- `factorial(n)/factorial(n-r)`, permutation
- `choose(n,r)`, combination
- `x = c(1, 2, ...)`, to input a vector
- `mean(x)`, mean
- `median(x)`, median
- `sd(x)`, standard deviation
- `cov(x,y)`, covariance
- `lm(y ~ x)`, least-squares regression line
- **Probability Distributions** "-" filled with d, p or q
  - `-binom(x,n,p)` binomial
  - `-gamma(x-1,p)` gamma
  - `-hyper(x,m,n,k)` hypergeometric
  - `-pois(x,mu)` Poisson

$f(x_i) = P(X = x_i)$
$F(x) = P(X \le x)$

General formula for continuous rv:

$$P(a < X \le b) = \int_a^b f(x)\,dx$$

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(w)\,dw$$

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\,dx \quad \text{and} \quad E[u(X)] = \int_{-\infty}^{\infty} u(x)\,f(x)\,dx$$

$$V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

Exponential distribution:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

$$E[X] = 1/\lambda \text{ and } V(X) = 1/\lambda^2$$

Gamma distribution:

$$f(y) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} y^{\alpha-1} e^{-y/\beta}, \ 0 \le y < \infty$$

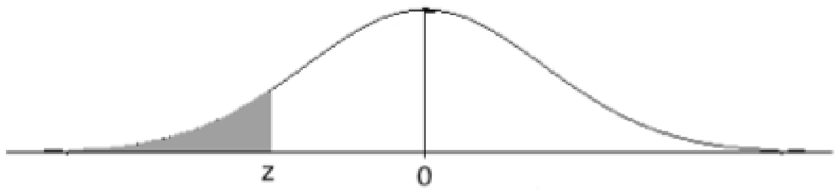
$$E[X] = \mu = \alpha\beta \text{ and } V(X) = \sigma^2 = \alpha\beta^2$$

R “base” commands for distributions:

“_” filled in with d, p or q

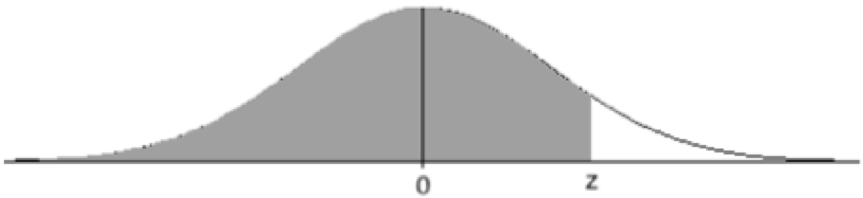
```
_exp( )
_gamma( )
_norm( )
```

Table of Standard Normal Probabilities for Negative Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table of Standard Normal Probabilities for Positive Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998