MATH 3339 Test 2 Formulas

$$f(x_i) = P(X = x_i)$$
$$F(x) = P(X \le x)$$

General formula for continuous rv:

$$P(a < X \le b) = \int_a^b f(x)\,dx$$
$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(w)\,dw$$
$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\,dx \quad \text{and} \quad E[u(X)] = \int_{-\infty}^{\infty} u(x) f(x)\,dx$$
$$V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

Exponential distribution:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$
$$E[X] = 1/\lambda \text{ and } V(X) = 1/\lambda^2$$

Gamma distribution:

$$f(y) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} y^{\alpha-1} e^{-y/\beta},\ 0 \le y < \infty$$
$$E[X] = \mu = \alpha\beta \text{ and } V(X) = \sigma^2 = \alpha\beta^2$$

R "base" commands for distributions:

"_" filled in with d, p or q

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_exp( )
_gamma( )
_norm( )
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Hypothesis tests:

Test	Null Hypothesis	Test Statistic
One-sample z-test for means	$\mu = \mu_o$	$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$
One-sample t-test for means	$\mu = \mu_o$	$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
One-sample z-test for proportions	$p = p_o$	$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$
Two-sample t-test for means	$\mu_1 = \mu_2$	$t = \frac{\bar{\bar{x}}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$
Paired t-test for means	$\mu_D = 0$	$t = \frac{\bar{x}_D - \mu_D}{s_D/\sqrt{n}}$
Two-sample z-test for proportions	$p_1 = p_2$	$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$

Confidence Intervals:

One-sample z-interval: $\bar{x} \pm z^* \left(\frac{\sigma}{\sqrt{n}}\right)$

One-sample t-interval: $\bar{x} \pm t^*_{n-1} \left(\frac{s}{\sqrt{n}}\right)$

One-proportion z-interval: $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

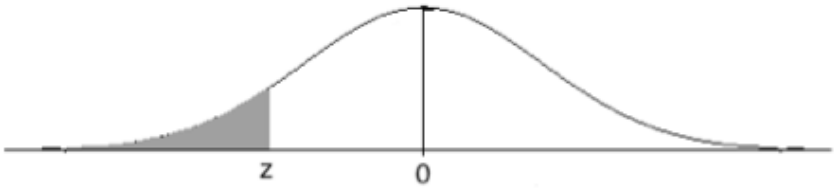
Two-sample t-interval: $(\bar{\bar{x}}_1 - \bar{x}_2) \pm t^*_\nu \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}, \nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^3}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$

Two-sample z-interval for proportions: $(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

Confidence interval for matched pairs: $\bar{x}_D \pm t^*_{n-1} \left(\frac{s_D}{\sqrt{n}}\right)$, where D = differences of the pairs.

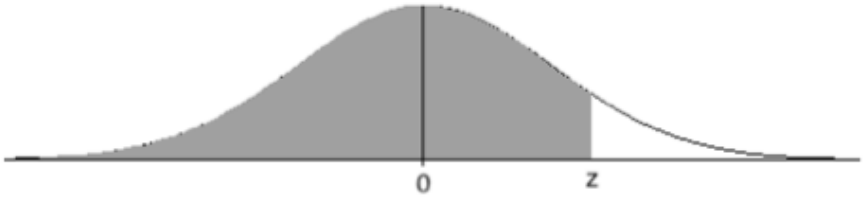
Confidence interval for variance: $\frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}, n-1}}$

Table of Standard Normal Probabilities for Negative Z-scores



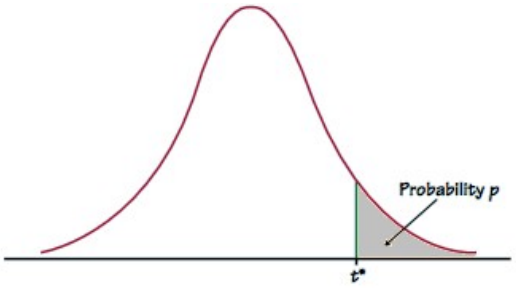
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table of Standard Normal Probabilities for Positive Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table entry for p and C is the critical value t* with probability p lying to its right and probability C lying between –t* and t*



	Upper tail probability *p*											
df	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.02**	**0.01**	**0.005**	**0.0025**	**0.001**	**0.0005**
1	1.000	1.376	1.963	3.078	6.314	12.706	15.895	31.821	63.657	127.321	318.309	636.619
2	0.8165	1.0607	1.3862	1.8856	2.9200	4.3027	4.8487	6.9646	9.9248	14.0890	22.3271	31.5991
3	0.7649	0.9785	1.2498	1.6377	2.3534	3.1824	3.4819	4.5407	5.8409	7.4533	10.2145	12.9240
4	0.7407	0.9410	1.1896	1.5332	2.1318	2.7764	2.9985	3.7469	4.6041	5.5976	7.1732	8.6103
5	0.7267	0.9195	1.1558	1.4759	2.0150	2.5706	2.7565	3.3649	4.0321	4.7733	5.8934	6.8688
6	0.7176	0.9057	1.1342	1.4398	1.9432	2.4469	2.6122	3.1427	3.7074	4.3168	5.2076	5.9588
7	0.7111	0.8960	1.1192	1.4149	1.8946	2.3646	2.5168	2.9980	3.4995	4.0293	4.7853	5.4079
8	0.7064	0.8889	1.1081	1.3968	1.8595	2.3060	2.4490	2.8965	3.3554	3.8325	4.5008	5.0413
9	0.7027	0.8834	1.0997	1.3830	1.8331	2.2622	2.3984	2.8214	3.2498	3.6897	4.2968	4.7809
10	0.6998	0.8791	1.0931	1.3722	1.8125	2.2281	2.3593	2.7638	3.1693	3.5814	4.1437	4.5869
11	0.6974	0.8755	1.0877	1.3634	1.7959	2.2010	2.3281	2.7181	3.1058	3.4966	4.0247	4.4370
12	0.6955	0.8726	1.0832	1.3562	1.7823	2.1788	2.3027	2.6810	3.0545	3.4284	3.9296	4.3178
13	0.6938	0.8702	1.0795	1.3502	1.7709	2.1604	2.2816	2.6503	3.0123	3.3725	3.8520	4.2208
14	0.6924	0.8681	1.0763	1.3450	1.7613	2.1448	2.2638	2.6245	2.9768	3.3257	3.7874	4.1405
15	0.6912	0.8662	1.0735	1.3406	1.7531	2.1314	2.2485	2.6025	2.9467	3.2860	3.7328	4.0728
16	0.6901	0.8647	1.0711	1.3368	1.7459	2.1199	2.2354	2.5835	2.9208	3.2520	3.6862	4.0150
17	0.6892	0.8633	1.0690	1.3334	1.7396	2.1098	2.2238	2.5669	2.8982	3.2224	3.6458	3.9651
18	0.6884	0.8620	1.0672	1.3304	1.7341	2.1009	2.2137	2.5524	2.8784	3.1966	3.6105	3.9216
19	0.6876	0.8610	1.0655	1.3277	1.7291	2.0930	2.2047	2.5395	2.8609	3.1737	3.5794	3.8834
20	0.6870	0.8600	1.0640	1.3253	1.7247	2.0860	2.1967	2.5280	2.8453	3.1534	3.5518	3.8495
21	0.6864	0.8591	1.0627	1.3232	1.7207	2.0796	2.1894	2.5176	2.8314	3.1352	3.5272	3.8193
22	0.6858	0.8583	1.0614	1.3212	1.7171	2.0739	2.1829	2.5083	2.8188	3.1188	3.5050	3.7921
23	0.6853	0.8575	1.0603	1.3195	1.7139	2.0687	2.1770	2.4999	2.8073	3.1040	3.4850	3.7676
24	0.6848	0.8569	1.0593	1.3178	1.7109	2.0639	2.1715	2.4922	2.7969	3.0905	3.4668	3.7454
25	0.6844	0.8562	1.0584	1.3163	1.7081	2.0595	2.1666	2.4851	2.7874	3.0782	3.4502	3.7251
26	0.6840	0.8557	1.0575	1.3150	1.7056	2.0555	2.1620	2.4786	2.7787	3.0669	3.4350	3.7066
27	0.6837	0.8551	1.0567	1.3137	1.7033	2.0518	2.1578	2.4727	2.7707	3.0565	3.4210	3.6896
28	0.6834	0.8546	1.0560	1.3125	1.7011	2.0484	2.1539	2.4671	2.7633	3.0469	3.4082	3.6739
29	0.6830	0.8542	1.0553	1.3114	1.6991	2.0452	2.1503	2.4620	2.7564	3.0380	3.3962	3.6594
30	0.6828	0.8538	1.0547	1.3104	1.6973	2.0423	2.1470	2.4573	2.7500	3.0298	3.3852	3.6460
40	0.6807	0.8507	1.0500	1.3031	1.6839	2.0211	2.1229	2.4233	2.7045	2.9712	3.3069	3.5510
50	0.6794	0.8489	1.0473	1.2987	1.6759	2.0086	2.1087	2.4033	2.6778	2.9370	3.2614	3.4960
60	0.6786	0.8477	1.0455	1.2958	1.6706	2.0003	2.0994	2.3901	2.6603	2.9146	3.2317	3.4602
80	0.6776	0.8461	1.0432	1.2922	1.6641	1.9901	2.0878	2.3739	2.6387	2.8870	3.1953	3.4163
100	0.6770	0.8452	1.0418	1.2901	1.6602	1.9840	2.0809	2.3642	2.6259	2.8707	3.1737	3.3905
1000	0.6747	0.8420	1.0370	1.2824	1.6464	1.9623	2.0564	2.3301	2.5808	2.8133	3.0984	3.3003
z*	0.6745	0.8416	1.0364	1.2816	1.6449	1.9600	2.0537	2.3263	2.5758	2.8070	3.0902	3.2905
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

Lower Tail Probability

df	0.01	0.02	0.025	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.75	0.8	0.85	0.9	0.95	0.975	0.98	0.99
1	0.0002	0.0006	0.001	0.0039	0.0158	0.0358	0.0642	0.1015	0.1485	0.275	0.4549	0.7083	1.0742	1.3233	1.6424	2.0723	2.7055	3.8415	5.0239	5.4119	6.6349
2	0.0201	0.0404	0.0506	0.1026	0.2107	0.325	0.4463	0.5754	0.7133	1.0217	1.3863	1.8326	2.4079	2.7726	3.2189	3.7942	4.6052	5.9915	7.3778	7.824	9.2103
3	0.1148	0.1848	0.2158	0.3518	0.5844	0.7978	1.0052	1.2125	1.4237	1.8692	2.366	2.9462	3.6649	4.1083	4.6416	5.317	6.2514	7.8147	9.3484	9.8374	11.3449
4	0.2971	0.4294	0.4844	0.7107	1.0636	1.3665	1.6488	1.9226	2.1947	2.7528	3.3567	4.0446	4.8784	5.3853	5.9886	6.7449	7.7794	9.4877	11.1433	11.6678	13.2767
5	0.5543	0.7519	0.8312	1.1455	1.6103	1.9938	2.3425	2.6746	2.9999	3.6555	4.3515	5.1319	6.0644	6.6257	7.2893	8.1152	9.2364	11.0705	12.8325	13.3882	15.0863
6	0.8721	1.1344	1.2373	1.6354	2.2041	2.6613	3.0701	3.4546	3.8276	4.5702	5.3481	6.2108	7.2311	7.8408	8.5581	9.4461	10.6446	12.5916	14.4494	15.0332	16.8119
7	1.239	1.5643	1.6899	2.1673	2.8331	3.3583	3.8223	4.2549	4.6713	5.4932	6.3458	7.2832	8.3834	9.0371	9.8032	10.7479	12.017	14.0671	16.0128	16.6224	18.4753
8	1.6465	2.0325	2.1797	2.7326	3.4895	4.0782	4.5936	5.0706	5.5274	6.4226	7.3441	8.3505	9.5245	10.2189	11.0301	12.0271	13.3616	15.5073	17.5345	18.1682	20.0902
9	2.0879	2.5324	2.7004	3.3251	4.1682	4.8165	5.3801	5.8988	6.3933	7.357	8.3428	9.4136	10.6564	11.3888	12.2421	13.288	14.6837	16.919	19.0228	19.679	21.666
10	2.5582	3.0591	3.247	3.9403	4.8652	5.5701	6.1791	6.7372	7.2672	8.2955	9.3418	10.4732	11.7807	12.5489	13.442	14.5339	15.9872	18.307	20.4832	21.1608	23.2093
11	3.0535	3.6087	3.8157	4.5748	5.5778	6.3364	6.9887	7.5841	8.1479	9.2373	10.341	11.5298	12.8987	13.7007	14.6314	15.7671	17.275	19.6751	21.92	22.6179	24.725
12	3.5706	4.1783	4.4038	5.226	6.3038	7.1138	7.8073	8.4384	9.0343	10.182	11.3403	12.5838	14.0111	14.8454	15.812	16.9893	18.5493	21.0261	23.3367	24.054	26.217
13	4.1069	4.7654	5.0088	5.8919	7.0415	7.9008	8.6339	9.2991	9.9257	11.1291	12.3398	13.6356	15.1187	15.9839	16.9848	18.202	19.8119	22.362	24.7356	25.4715	27.6882
14	4.6604	5.3682	5.6287	6.5706	7.7895	8.6963	9.4673	10.1653	10.8215	12.0785	13.3393	14.6853	16.2221	17.1169	18.1508	19.4062	21.0641	23.6848	26.1189	26.8728	29.1412
15	5.2293	5.9849	6.2621	7.2609	8.5468	9.4993	10.307	11.0365	11.7212	13.0297	14.3389	15.7332	17.3217	18.2451	19.3107	20.603	22.3071	24.9958	27.4884	28.2595	30.5779
16	5.8122	6.6142	6.9077	7.9616	9.3122	10.309	11.1521	11.9122	12.6243	13.9827	15.3385	16.7795	18.4179	19.3689	20.4651	21.7931	23.5418	26.2962	28.8454	29.6332	31.9999
17	6.4078	7.255	7.5642	8.6718	10.0852	11.1249	12.0023	12.7919	13.5307	14.9373	16.3382	17.8244	19.511	20.4887	21.6146	22.977	24.769	27.5871	30.191	30.995	33.4087
18	7.0149	7.9062	8.2307	9.3905	10.8649	11.9463	12.857	13.6753	14.4399	15.8932	17.3379	18.8679	20.6014	21.6049	22.7595	24.1555	25.9894	28.8693	31.5264	32.3462	34.8053
19	7.6327	8.567	8.9065	10.117	11.6509	12.7727	13.7158	14.562	15.3517	16.8504	18.3377	19.9102	21.6891	22.7178	23.9004	25.3289	27.2036	30.1435	32.8523	33.6874	36.1909
20	8.2604	9.2367	9.5908	10.8508	12.4426	13.6039	14.5784	15.4518	16.2659	17.8088	19.3374	20.9514	22.7745	23.8277	25.0375	26.4976	28.412	31.4104	34.1696	35.0196	37.5662
21	8.8972	9.9146	10.2829	11.5913	13.2396	14.4393	15.4446	16.3444	17.1823	18.7683	20.3372	21.9915	23.8578	24.9348	26.1711	27.662	29.6151	32.6706	35.4789	36.3434	38.9322
22	9.5425	10.6	10.9823	12.338	14.0415	15.2788	16.314	17.2396	18.1007	19.7288	21.337	23.0307	24.939	26.0393	27.3015	28.8225	30.8133	33.9244	36.7807	37.6595	40.2894
23	10.1957	11.2926	11.6886	13.0905	14.848	16.1219	17.1865	18.1373	19.0211	20.6902	22.3369	24.0689	26.0184	27.1413	28.4288	29.9792	32.0069	35.1725	38.0756	38.9683	41.6384
24	10.8564	11.9918	12.4012	13.8484	15.6587	16.9686	18.0618	19.0373	19.9432	21.6525	23.3367	25.1063	27.096	28.2412	29.5533	31.1325	33.1962	36.415	39.3641	40.2704	42.9798
25	11.524	12.6973	13.1197	14.6114	16.4734	17.8184	18.9398	19.9393	20.867	22.6156	24.3366	26.143	28.1719	29.3389	30.6752	32.2825	34.3816	37.6525	40.6465	41.5661	44.3141
26	12.1981	13.4086	13.8439	15.3792	17.2919	18.6714	19.8202	20.8434	21.7924	23.5794	25.3365	27.1789	29.2463	30.4346	31.7946	33.4295	35.5632	38.8851	41.9232	42.8558	45.6417
27	12.8785	14.1254	14.5734	16.1514	18.1139	19.5272	20.703	21.7494	22.7192	24.544	26.3363	28.2141	30.3193	31.5284	32.9117	34.5736	36.7412	40.1133	43.1945	44.14	46.9629
28	13.5647	14.8475	15.3079	16.9279	18.9392	20.3857	21.588	22.6572	23.6475	25.5093	27.3362	29.2486	31.3909	32.6205	34.0266	35.715	37.9159	41.3371	44.4608	45.4188	48.2782
29	14.2565	15.5745	16.0471	17.7084	19.7677	21.2468	22.4751	23.5666	24.577	26.4751	28.3361	30.2825	32.4612	33.7109	35.1394	36.8538	39.0875	42.557	45.7223	46.6927	49.5879
30	14.9535	16.3062	16.7908	18.4927	20.5992	22.1103	23.3641	24.4776	25.5078	27.4416	29.336	31.3159	33.5302	34.7997	36.2502	37.9903	40.256	43.773	46.9792	47.9618	50.8922
40	22.1643	23.8376	24.433	26.5093	29.0505	30.8563	32.345	33.6603	34.8719	37.134	39.3353	41.6222	44.1649	45.616	47.2685	49.2439	51.8051	55.7585	59.3417	60.4361	63.6907
50	29.7067	31.6639	32.3574	34.7643	37.6886	39.7539	41.4492	42.9421	44.3133	46.8638	49.3349	51.8916	54.7228	56.3336	58.1638	60.346	63.1671	67.5048	71.4202	72.6133	76.1539
60	37.4849	39.6994	40.4817	43.188	46.4589	48.7587	50.6406	52.2938	53.8091	56.62	59.3347	62.1348	65.2265	66.9815	68.9721	71.3411	74.397	79.0819	83.2977	84.5799	88.3794
70	45.4417	47.8934	48.7576	51.7393	55.3289	57.8443	59.8978	61.6983	63.346	66.3961	69.3345	72.3583	75.6893	77.5767	79.7146	82.2554	85.527	90.5312	95.0232	96.3875	100.4252
80	53.5401	56.2128	57.1532	60.3915	64.2778	66.9938	69.2069	71.1445	72.9153	76.1879	79.3343	82.5663	86.1197	88.1303	90.4053	93.1058	96.5782	101.8795	106.6286	108.0693	112.3288
90	61.7541	64.6347	65.6466	69.126	73.2911	76.1954	78.5584	80.6247	82.5111	85.9925	89.3342	92.7614	96.5238	98.6499	101.0537	103.9041	107.565	113.1453	118.1359	119.6485	124.1163
100	70.0649	73.1422	74.2219	77.9295	82.3581	85.4406	87.9453	90.1332	92.1289	95.8078	99.3341	102.9459	106.9058	109.1412	111.6667	114.6588	118.498	124.3421	129.5612	131.1417	135.8067